REMARKS

In the Office Action of July 19, 2006, claims 1-8 were rejected under 37 U.S.C. 117, second paragraph as indefinite and under 35 U.S.C 101 as directed to non-statutory subject matter.

I. Indefiniteness

Applicant respectfully traverses the rejection of claims 1-8 as indefinite because the claims "point to variables that are not explicitly defined, in terms of what they represent and how they are calculated."

In claim 1, the step of "constructing an exponential moving average operator" is criticized as indefinite. As pointed out on page 4 of the application, an operator is a mapping, an important example of an operator is a moving average operator and a simple example of a moving average operator is an exponential moving average (EMA) operator. The EMA operator is a well known operator described, for example, at pages 370-372 of J.C. Hull, Options, Futures & Other Derivatives (4th. ed., Prentice Hall, 2000). Copies of pages 368-372 are enclosed for the Examiner's convenience. The Hull textbook is one of the standard textbooks in this field and is cited at page 16, lines 25-26 of the specification. Since the EMA operator is well known to those skilled in the art, and is described in detail in the application on pages 4 to 6, it is respectfully submitted that the step of "constructing an exponential moving average operator" is definite.

As the Examiner acknowledges, claim 3 provides details of the exponential moving operator but the Examiner faults claim 3 because it recites that one element (ν) of the equation set forth in claim 3 "depends on a chosen interpolation scheme." The equation set forth in claim 3 is an interactive formula for calculating the EMA of discrete time series values $z(t_i)$ at

irregularly spaced time points t_i. This equation is presented in slightly different format in equation (6) on page 5 of the specification.

As explained on page 5 beginning at line 20 it is necessary to make an assumption about the behavior of z(t) between the discrete time points t_i because the discrete data is being used in a convolution defined in continuous time. In other words, it is necessary to make an interpolation. This behavioral assumption is incorporated into claim 3 and equation 6 by the value ν. As set forth at page 5, equation 8 and lines 22 to 24, if we assume linear interpolation between points, then $\nu = (1 - \mu)/\alpha$. If, however we assume that the value of the old tick (e.g. $z(t_{n-1})$) remains constant through the whole interval before the next tick (e.g. $z(t_n)$), then $\nu = 1$ as set forth in equation (9). This has the effect of eliminating from the equation of claim 3 the last term Z_n which is the value of the new tick.

Alternatively, as discussed at page 5, line 28 to page 6, line 2, an assumption can be made that interpolation does not apply, in which case $\nu = \mu$ as set forth in equation (10). This has the effect of eliminating from the equation of claim 3 the middle term.

In view of the description of ν at pages 4 to 6 of the specification, it is submitted that the equation set forth in claim 3 is definite and that the description of ν as being dependent on the chosen interpolation procedure is definite and appropriate. Claim 3 has been amended slightly to clarify that ν is a value that depends on the chosen interpolation procedure. New claims 9, 10 and 11 have been added specifying the explicit value of ν for the three cases described above. Claim 3 has also been amended to conform the value of α to the value set forth in equation (7) of the specification.

II. Nonstatutory Subject Matter

In rejecting claims 1-8 under 35 U.S.C. § 101 as directed to non-statutory subject matter, the examiner relies on two grounds: the invention must accomplish a practical application and produce a concrete result. In requiring the invention to produce a practical application, the Examiner requires that it produce "a useful, concrete and tangible result", citing State Street Bank v Signature Financial, 149 F.2d 1368, 1373, 47 USPQ2d 1596, 1601-02 (Fed Cir. 1998). The applicant respectfully traverse this rejection.

It is evident that the two grounds for rejection are really only one. The concrete result requirement of the second ground is found in the "useful, concrete and tangible result" language relied on for the first ground of rejection.

The applicable case law is In re Alappat, 33 F.3d 1526, 31 USPQ 2d 1545 (Fed. Cir. 1994); State Street Bank v. Signature Financial, 149 F.3d 1368, 47 USPQ 2d 1596 (Fed. Cir. 1998); and AT&T Corp. v. Excel Communications, Inc., 172 F.3d 1352, 50 USPQ 2d 1447 (Fed. Cir. 1999). Each of these decisions found that the claims at issue satisfied the requirements of 35 U.S.C. 101 because they produced a "useful, concrete and tangible result." However, none of these decisions require that inventions posses a certain level of 'real world' value" or "guarantee a concrete result" as the Examiner seems to require.

Applicants submit that Alappat, State Street, and AT&T equate the requirement of "useful, concrete and tangible result" to the utility requirement of 35 U.S.C. 101 and that applicants' invention meets this requirement as well. In Alappat the invention at issue was apparatus for creating a smooth waveform display. It achieved this result by an interpolation process. The Federal Circuit reversed the Board's affirmance of the Examiner's rejection of the claim:

"This is not a disembodied mathematical concept which may be characterized as an 'abstract idea,' but rather a specific machine to produce a useful, concrete and tangible result."

"The fact that the four claimed means elements function to transform one set of data to another through what may be viewed as a series of mathematical calculations does not alone justify a holding that the claim as a whole is directed to nonstatutory subject matter." 33 F.3d at 1544, 31 USPQ 2d at 1557.

Similarly, in State Street the claim at issue was a system for calculating a share price for certain investment funds. Again, the Federal Circuit found this system to constitute patentable subject matter:

"Today, we hold that the transformation of data, representing discrete dollar amounts, by a machine through a series of mathematical calculations into a final share price, constitutes a practical application of a mathematical algorithm, formula, or calculation, because it produces 'a useful, concrete and tangible result'—a final share price momentarily fixed for recording and reporting purposes and even accepted and relied upon by regulatory authorities and in subsequent trades." 47 USPQ 2d at 1601.

And in AT&T, the claim at issue was a method claim for generating a message record that included a primary interexchange carrier (PIC) indicator that was used to facilitate differential billing. While the PIC indicator was generated using simple mathematical principles (Boolean arithmetic), the Federal Circuit found the method to constitute patentable subject matter:

"The PIC indicator represents information about the call recipient's PIC, a useful, non-abstract result that facilitates differential billing of long-distance calls made by an IXC's

subscriber. Because the claimed process applies the Boolean principle to produce a useful, concrete, tangible result without pre-empting other uses of the mathematical principle, on its face the claimed process comfortable falls within the scope of § 101." 50 USPQ 2d at 1452.

Just as the systems and method of Alappat, State Street and AT&T are statutory because they produced a "useful, concrete, and tangible result," so too the claims of the present application are statutory under 35 U.S.C. § 101 because they recite a method that produces a "useful, concrete and tangible result." Specifically, the invention is a method for determining value at risk. As set forth at page 1, lines 12-17 risk measurement is critical in the operation of banks and other financial institutions since risk levels determine capital requirements. Applicant has found that conventional methods of measuring risk by analyzing daily data are inadequate and applicant proposes a new method that is the subject of the present claims.

There can be no doubt that applicant's method has practical utility because the amount of risk that is measured determines the capital requirements of the financial institution. As pointed out at page 2, lines 16-20, applicant has observed that conventional methods of measuring risk by analyzing daily data are very sensitive to the choice of sampling time and have high levels of stochastic (or random) noise. Applicant's method for calculating risk has significantly reduced noise and up-to-date results are available at each tick.

Since applicant's invention does have practical utility, it is respectfully submitted that applicant's claims meet the requirement that they produce a "useful, concrete and tangible result" and therefore are statutory under 35 U.S.C. 101.

For the reasons set forth above, applicant submits that there is no separate requirement that the claimed invention guarantee a concrete result as the Examiner seems to require. Applicant notes that the Examiner has cited no case law in support of his position. Applicant

further notes that the Examiner relies primarily on applicant's discussion of the use of different interpolation formulas at page 5, lines 20-30 and page 6 lines 28-30 for support of the Examiner's portion. As emphasized above, the particular interpolation formula to use is a matter of choice. Once the choice is made, repeated application of the formula to the same set of data will produce the same results.

The claims of the application recite applicants' method of determining value-at-risk. Just as the system of State Street for calculating a share price was found to produce a useful, concrete and tangible result, it is respectfully submitted that the claims of this application produce a useful, concrete, and tangible result.

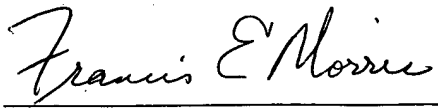
In conclusion, since the claims produce a useful, concrete and tangible result, the rejection of claims 1-8 under 35 U.S.C. 101 should be withdrawn.

Aside from the fee for an extension of time, no additional fee is believed to be due for filing this response. However, if a fee is due, please charge such fee to Morgan, Lewis & Bockius LLP Deposit Account No. 50-0310.

If the Examiner believes a telephone interview would expedite prosecution of this application, he is invited to call applicant's attorney at the number given below.

Respectfully submitted,

Date December 15, 2006



Francis E. Morris
Reg. No. 24,615
Morgan, Lewis & Bockius LLP
Customer No. 009629
(212) 309-6632

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John C. Hull
Joseph L. Rotman School of Management
University of Toronto

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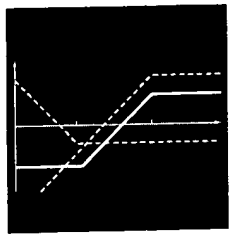
CHAPTER

15



ESTIMATING VOLATILITIES AND CORRELATIONS

This chapter explains ways that historical data can be used to produce estimates of the current levels of volatilities and correlations, as well as forecasts of the future values of these variables. The chapter is relevant to both the calculation of value at risk and the valuation of derivatives. When calculating value at risk, we are most interested in the current levels of volatilities and correlations because we are assessing possible changes in the value of a portfolio over a very short period of time. When valuing derivatives, forecasts of volatilities and correlations over the whole life of the derivative are usually required.

The chapter considers models with imposing names such as exponentially weighted moving average (EWMA), autoregressive conditional heteroscedasticity (ARCH), and generalized autoregressive conditional heteroscedasticity (GARCH). The distinctive feature of the models is that they recognize that volatilities and correlations are not constant. During some periods, a particular volatility or correlation may be relatively low whereas during other periods it may be relatively high. The models attempt to keep track of the variations in the volatility or correlation through time.

15.1 ESTIMATING VOLATILITY

Define σ_n as the volatility of a market variable on day n, as estimated at the end of day $n-1$. The square of the volatility on day n, σ_n^2, is the *variance rate*.

The standard approach to estimating σ_n from historical data was described in section 11.3. Suppose that the value of the market variable at the end of day i is S_i. The variable u_i is defined as the continuously compounded return during day i (between the end of day $i-1$ and the end of day i):

$$u_i = \ln \frac{S_i}{S_{i-1}}$$

An unbiased estimate of the variance rate per day, σ_n^2, using the most recent m observations on the u_i is

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}(u_{n-i} - \overline{u})^2 \tag{15.1}$$

where $\overline{u}$ is the mean of the u_i's:

$$\overline{u} = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}$$

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For the purposes of calculating VaR, the formula in equation (15.1) is usually changed in a number of ways:

1. u_i is defined as the proportional change in the market variable between the end of day $i-1$ and the end of day i so that:[1]

$$u_i = \frac{S_i - S_{i-1}}{S_{i-1}} \tag{15.2}$$

2. $\overline{u}$ is assumed to be zero.[2]
3. $m-1$ is replaced by m.[3]

These three changes make very little difference to the variance estimates that are calculated. The formula for variance rate becomes

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2 \tag{15.3}$$

where u_i is given by equation (15.2).[4]

Weighting Schemes

Equation (15.3) gives equal weight to all u_i^2's. Given that the objective is to monitor the current level of volatility, it is appropriate to give more weight to recent data. A model that does this is

$$\sigma_n^2 = \sum_{i=1}^{m} \alpha_i u_{n-i}^2 \tag{15.4}$$

[1]This is consistent with the point made in section 14.1 about the way that volatility is defined for the purposes of VaR calculations.

[2]As explained in section 14.2, this assumption usually has very little effect on estimates of the variance because the expected change in a variable in one day is very small when compared with the standard deviation of changes. As an alternative to the assumption we can define u_i as the realized return minus the expected return on day i.

[3]Replacing $m-1$ by m moves us from an unbiased estimate of the variance to a maximum likelihood estimate. Maximum likelihood estimates are discussed later in the chapter.

[4]Note that the u's in this chapter play the same role as the Δx's in chapter 14. Both are daily proportional changes in market variables. In the case of the u's, the subscripts count observations made on different days on the same market variable. In the case of the Δx's, they count observations made on the same day on different market variables. The use of subscripts for σ is similarly different between the two chapters. In this chapter, the subscripts refer to days; in chapter 14 they referred to market variables.

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The variable α_i is the amount of weight given to the observation i days ago. The α's are positive. Because we wish to give less weight to older observations, $\alpha_i < \alpha_j$ when $i > j$. The weights must sum to unity so that:

$$\sum_{i=1}^{m} \alpha_i = 1$$

An extension of the idea in equation (15.4) is to assume that there is a long-run average volatility and that this should be given some weight. This leads to a model that takes the form

$$\sigma_n^2 = \gamma V + \sum_{i=1}^{m} \alpha_i u_{n-i}^2 \tag{15.5}$$

where V is the long-run volatility and γ is the weight assigned to V. Because the weights must sum to unity:

$$\gamma + \sum_{i=1}^{m} \alpha_i = 1$$

This is known as an ARCH(m) model. It was first suggested by Engle.[5] The estimate of the variance is based on a long-run average variance and m observations. The older an observation, the less weight it is given. Defining $\omega = \gamma V$, the model in equation (15.5) can be written

$$\sigma_n^2 = \omega + \sum_{i=1}^{m} \alpha_i u_{n-i}^2 \tag{15.6}$$

This is the version of the model that is used when parameters are estimated.

In the next two sections we discuss two important approaches to monitoring volatility using the ideas in equations (15.4) and (15.5).

15.2 THE EXPONENTIALLY WEIGHTED MOVING AVERAGE MODEL

The exponentially weighted moving average (EWMA) model is a particular case of the model in equation (15.4) where the weights, α_i, decrease exponentially as we move back through time. Specifically, $\alpha_{i+1} = \lambda\alpha_i$ where λ is a constant between zero and one.

It turns out that this weighting scheme leads to a particularly simple formula for updating volatility estimates. The formula is

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1 - \lambda)u_{n-1}^2 \tag{15.7}$$

[5] See R. Engle, "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of UK Inflation," *Econometrica*, 50 (1982), 987–1008.

The estimate, σ_n, of the volatility for day n (made at the end of day $n-1$) is calculated from σ_{n-1} (the estimate that was made one day ago of the volatility for day $n-1$) and u_{n-1} (the most recent observation on changes in the market variable).

To understand why equation (15.7) corresponds to weights that decrease exponentially, we substitute for σ_{n-1}^2 to get

$$\sigma_n^2 = \lambda[\lambda\sigma_{n-2}^2 + (1-\lambda)u_{n-2}^2] + (1-\lambda)u_{n-1}^2$$

or

$$\sigma_n^2 = (1-\lambda)(u_{n-1}^2 + \lambda u_{n-2}^2) + \lambda^2\sigma_{n-2}^2$$

Substituting in a similar way for σ_{n-2}^2 gives

$$\sigma_n^2 = (1-\lambda)(u_{n-1}^2 + \lambda u_{n-2}^2 + \lambda^2 u_{n-3}^2) + \lambda^3\sigma_{n-3}^2$$

Continuing in this way, we see that

$$\sigma_n^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}u_{n-i}^2 + \lambda^m\sigma_0^2$$

For a large m, the term $\lambda^m\sigma_0^2$ is sufficiently small to be ignored so that equation (15.7) is the same as equation (15.4) with $\alpha_i = (1-\lambda)\lambda^{i-1}$. The weights for the u_i's decline at rate λ as we move back through time. Each weight is λ times the previous weight.

Example 15.1 Suppose that λ is 0.90, the volatility estimated for day $n-1$ is 1% per day, and the proportional change in the market variable during day $n-1$ is 2%. This means that $\sigma_{n-1}^2 = 0.01^2 = 0.0001$ and $u_{n-1}^2 = 0.02^2 = 0.0004$. Equation (15.7) gives

$$\sigma_n^2 = 0.9 \times 0.0001 + 0.1 \times 0.0004 = 0.00013$$

The estimate of the volatility for day n, σ_n, is, therefore, $\sqrt{0.00013}$ or 1.14% per day. Note that the expected value of u_{n-1}^2 is σ_{n-1}^2 or 0.0001. In this example, the realized value of u_{n-1}^2 is greater than the expected value, and as a result our volatility estimate increases. If the realized value of u_{n-1}^2 had been less than its expected value, our estimate of the volatility would have decreased.

The EWMA approach has the attractive feature that relatively little data need to be stored. At any given time, we need to remember only the current estimate of the variance rate and the most recent observation on the value of the market variable. When we get a new observation on the value of the market variable, we calculate a new u^2 and use equation (15.7) to update our estimate of the variance rate. The old estimate of the variance rate and the old value of the market variable can then be discarded.

The EWMA approach is designed to track changes in the volatility. Suppose there is a big move in the market variable on day $n-1$ so that u_{n-1}^2 is large. From equation (15.7) this causes σ_n, our estimate of the daily volatility for day n, to move upward. The value of λ governs how responsive the estimate of the daily volatility is to the most recent observations on the u_i^2's. A low value of λ leads to a great deal of weight being given to the u_{n-1}^2 when σ_n is calculated. In this case, the estimates produced for the volatility on successive days are themselves highly volatile. A high

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value of λ (i.e., a value close to 1.0) produces estimates of the daily volatility that respond relatively slowly to new information provided by the u_i^2.

J. P. Morgan uses the EWMA model with $\lambda = 0.94$ for updating daily volatility estimates in its RiskMetrics database.[6] The company found that, across a range of different market variables, this value of λ gives forecasts of the variance rate that come closest to the realized variance rate.[7] The realized variance rate on a particular day was calculated as an equally weighted average of the u_i^2 on the subsequent 25 days.

15.3 THE GARCH (1,1) MODEL

We now move on to discuss what is known as the GARCH (1,1) model proposed by Bollerslev in 1986.[8] The difference between the GARCH (1,1) model and the EWMA model is analogous to the difference between equation (15.4) and equation (15.5). In GARCH (1,1), σ_n^2 is calculated from a long-run average variance rate, V, as well as from σ_{n-1} and u_{n-1}. The equation for GARCH (1,1) is

$$\sigma_n^2 = \gamma V + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{15.8}$$

where γ is the weight assigned to V, α is the weight assigned to u_{n-1}^2, and β is the weight assigned to σ_{n-1}^2. Because the weights must sum to one:

$$\gamma + \alpha + \beta = 1$$

The EWMA model is a particular case of GARCH (1,1) where $\gamma = 0$, $\alpha = 1 - \lambda$, and $\beta = \lambda$.

The "(1,1)" in GARCH (1,1) indicates that σ_n^2 is based on the most recent observation of u^2 and the most recent estimate of the variance rate. The more general GARCH (p, q) model calculates σ_n^2 from the most recent p observations on u^2 and the most recent q estimates of the variance rate.[9] GARCH (1,1) is by far the most popular of the GARCH models.

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[6] It is worth noting that the J. P. Morgan RiskMetrics database uses a different definition of volatility per day from the standard one. Its definition of volatility is 1.65 times the standard definition. From the tables at the end of this book, $N(-1.65) = 0.05$. Assuming a normal distribution, the RiskMetrics volatility can, therefore, be interpreted as the decline in a market variable that is expected to be exceeded on only 5% of days.

[7] See J. P. Morgan, *RiskMetrics Monitor*, Fourth Quarter, 1995. We will explain an alternative (maximum likelihood) approach to estimating parameters later in the chapter.

[8] See T. Bollerslev, "Generalized Autoregressive Conditional Heteroscedasticity," *Journal of Econometrics*, 31 (1986), 307–27.

[9] Other GARCH models have been proposed that incorporate asymmetric news. These models are designed so that σ_n depends on the sign of u_{n-1}. Arguably, the models are more appropriate for equities than GARCH (1,1). The volatility of an equity's price tends to be inversely related to the price so that a negative u_{n-1} has a bigger effect on σ_n than the same positive u_{n-1}. (See chapter 17.) For a discussion of models for handling asymmetric news see D. Nelson, "Conditional Heteroscedasticity and Asset Returns; A New Approach," *Econometrica*, 59 (1990), 347–70 and R.F. Engle and V. Ng, "Measuring and Testing the Impact of News on Volatility," *Journal of Finance*, 48 (1993), 1,749–78.